# Parser Generation

- ***Main Problem**: given a grammar **G**, how to build a **top-down parser** or a **bottom-up parser** for it ?*
- ***parser** : a program that, given a sentence, reconstructs a derivation for that sentence ---- if done sucessfully, it "recognize" the sentence*
- *all parsers read their input **left-to-right**, but construct parse tree differently.*
- ***bottom-up parsers ---** construct the tree from leaves to root*

  shift-reduce, LR, SLR, LALR, operator precedence
- ***top-down parsers ---** construct the tree from root to leaves*

  recursive descent, predictive parsing, LL(1)

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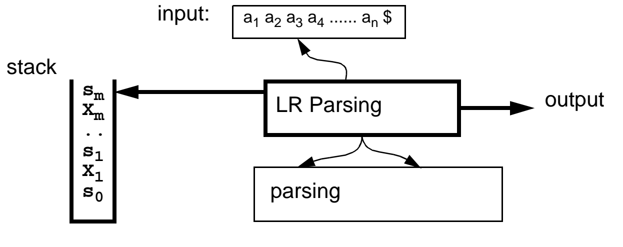
# Bottom-Up Parsing

- *Construct parse tree "bottom-up" --- from leaves to the root*
- *Bottom-up parsing always constructs **right-most derivation***
- *Important parsing algorithms: **shift-reduce**, **LR parsing***
- ***LR parser** components: input, stack (strings of grammar symbols and **states**), driver routine, parsing tables.*

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# LR Parsing

- *A sequence of new **state** symbols* $s_0, s_1, s_2, \ldots, s_m$ *----- each state sumarize the information contained in the stack below it.*
- *<u>Parsing configurations</u>: **(stack, remaining input)** written as*

  $(s_0X_1s_1X_2s_2\ldots X_ms_m\ ,\ a_ia_{i+1}a_{i+2}\ldots a_n\$)$

  next "move" is determined by $s_m$ and $a_i$
- *<u>Parsing tables</u>:* `ACTION[s,a]` *and* `GOTO[s,X]`

  *Table A* `ACTION[s,a]` --- `s` : *state*, `a` : *terminal*

  *its entries* (1) shift $s_k$ (2) reduce $A \to \beta$ (3) accept (4) error

  *Table G* `GOTO[s,X]` --- `s` : *state*, `X` : *non-terminal*
  *its entries are **states***

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# Constructing LR Parser

***How to construct the parsing table** `ACTION` **and** `GOTO` **?***

- ***basic idea**: first construct DFA to recognize handles, then use DFA to construct the parsing tables ! different parsing table yield different LR parsers **SLR(1)**, **LR(1)**, or **LALR(1)***
- ***augmented grammar** for context-free grammar **G** = **G**(T,N,P,S) is defined as **G' = G'**(T, N ∪ { S'}, P ∪ { S' -> S}, S') ------ adding non-terminal S' and the production S' -> S, and S' is the new start symbol. When S' -> S is reduced, parser accepts.*
- ***LR(0) item** for productions of a context-free grammar G ----- is a production with **dot** at some position in the r.h.s.*

  For $A \to XYZ$ , its items are $A \to .XYZ$  $A \to X.YZ$  $A \to XY.Z$  $A \to XYZ.$

  For $A \to \varepsilon$ , its items are just $A \to .$

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# LR(0) items and LR(0) DFA

- *Informally, item* `A -> X.YZ` *means a string derivable from* `X` *has been seen, and one from* `YZ` *is expected.* ***LR(0) items*** *are used as state names for LR(0) DFA or LR(0) NFA that recognizes* ***viable prefixes.***
- ***Viable prefixes*** *of a CFG are prefixes of right-sentential forms with no symbols to right of the* ***handle****; we can always add terminals on right to form a right-sentential form.*
- ***Two way to construct the LR(0) DFA:***
  1. *first construct LR(0) NFA and then convert it to a DFA !*
  2. *construct the LR(0) DFA directly !*
- ***From LR(0) DFA to the Parsing Table*** *-------- transition table for the DFA is the* **GOTO** *table; the states of DFA are states of the parser.*

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# Example: LR(0) Items

***CFG Grammar:***

```
E -> E + T | T
T -> T * F | F
F -> ( E ) | id
```

***Augmented Grammar:***

```
E' -> E
E  -> E + T | T
T  -> T * F | F
F  -> ( E ) | id
```

***LR(0) terms:***

| | | |
|---|---|---|
| E' -> . E | T -> . T * F | F -> ( E . ) |
| E' -> E . | T -> T . * F | F -> ( E ) . |
| E -> . E + T | T -> T * . F | F -> . id |
| E -> E . + T | T -> T * F . | F -> id . |
| E -> E + . T | T -> . F | |
| E -> E + T . | T -> F . | |
| E -> . T | F -> . ( E ) | |
| E -> T . | F -> ( . E ) | |

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# From LR(0) NFA to LR(0) DFA

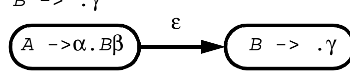
- *Construct* ***LR(0) NFA*** *with all LR(0) items of G as states, connect states by moving the* ***dot****;* ***final states*** *are those with dots at the end.*

  1. *for each item* $A \rightarrow \alpha.X\ \beta$

     $A \rightarrow \alpha.X\beta \xrightarrow{X} A \rightarrow \alpha X.\beta$

  2. *for each pair* $A \rightarrow \alpha.B\ \beta,\ B \rightarrow .\gamma$

     (expect to see a string derivable from $\gamma$)




- *Convert NFA to DFA using* ***subset construction algorithm.***
- *The states of the resulting* ***LR(0) DFA*** *---* $C = \{I_1, I_2, \ldots, I_n\}$ *are called* ***canonical LR(0) collection*** *for grammar G'*
- ***Disadvantage****: the NFA is often huge, and converting from NFA to DFA is tedious and time-consuming.*

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# Building LR(0) DFA Directly

- *Instead of building DFA from NFA,* ***we can build the LR(0) DFA directly****.*
- *Given a set of LR(0) items* **I** *,* **CLOSURE(I)** *is defined as*

```
repeat
  for each item A -> α.Bβ in I and
      each production B -> γ
            add B -> .γ to I, if it's not in I
until I does not change
```

- **GOTO(I,X)** *is defined as*

  **CLOSURE(***all items* $A \rightarrow \alpha X.\beta$ *for each* $A \rightarrow \alpha.X\beta$ *in* **I)**

- ***Canonical LR(0) collection*** *is computed by the following procedure:*

```
I0 = CLOSURE({S' -> .S}) and C = {I0}
repeat
  for each I ∈ C and grammar symbol X
    T = GOTO(I,X); if T ≠ ∅ and T ∉ C then C = C ∪ { T };
until C does not change
```

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*Resulting* `LR(0) DFA:` `C` *is the set of states;* `GOTO` *is the transition table*






# Constructing SLR(1) Parsing Table

- *From the **LR(0) DFA**, we can construct the parsing table ----- **SLR(1) parsing table.** The parser based on SLR(1) parsing table is called **SLR(1) parser**. The **SLR(1) grammars** are those whose SLR(1) parsing table does **not** contain any conflicts.*
- ***Algorithm --- use*** `C = {I0,...,In}, GOTO, FOLLOW:`

1. If A -> α.aβ is in $I_i$ and GOTO($I_i$,a) = $I_j$ where a is a terminal, set ACTION[i,a] to "shift j".
2. If A -> α. is in $I_i$, set ACTION[i,a] to "reduce A -> α" for all terminal a in FOLLOW(A).
3. If S' -> S. is in $I_i$, set ACTION[i,$] to "accept"
4. If GOTO($I_i$,A) = $I_j$, set GOTO[i,A] = j
5. set all other entries to "error"
6. set initial state to be $I_i$ with S' -> .S






# Limitation of SLR(1) Parser

- *Unfortunately, many **unambiguous** grammars are not SLR(1) gammars*

```
S -> L = R | R          L  means "l-value"
L -> *R | id            R  means "r-value"
R -> L                  *  means "contents of"
```

***Canonical LR(0) collection ---***

```
I0 : S' -> .S        I3:  S -> R.         I6: S -> L=.R
     S -> .L=R                                R -> .L
     S -> .R         I4:  L -> *.R            L -> .*R
     L -> .*R             R -> .L             L -> .id
     L -> .id             L -> .*R
     R -> .L              L -> .id        I7: L -> *R.

I1 : S' -> S.        I5:  L -> id.        I8: R -> L.

I2 : S -> L.=R                            I9: S -> L=R.
     R -> L.           FOLLOW(R) = {=,...}
```

*state 2 has a shift/reduce conflict on* **"="** *: shift 6 or reduce* `R -`






# LR(1) Parsing

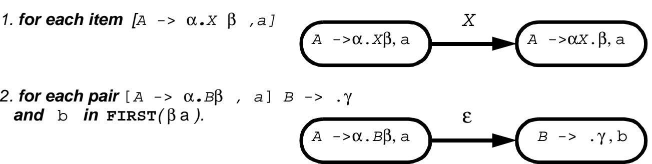
- *Conflict arises because **LR(0) states** do not encode enough left context --- in the previous example, reduction* `R -> L` *is wrong upon input* **=** *because "*`R = ...`*" **never** appears in right-sentential form.*
- ***Solution**: split LR(0) states by adding terminals to states, for example,* `[ A -> α. , a]` *results in reduction only if next symbol is* `a` *.*
- ***An LR(1) term** is in the form of* `[ A -> α.β , a ]` *where* `A -> αβ` *is a production and* `a` *is a terminal or* **$**
- *To build **LR(1) parsing table** --- we first build **LR(1) DFA** --- then construct the parsing table using the same SLR(1) algorithm except*

  *2. only if* `[A -> α. , a]` *is in* $I_i$ *,*
  *then set* **ACTION[i,a]** *to "reduce* `A-> α`*"*

- *To way to build **LR(1) DFA** ---- from NFA -> DFA or build DFA directly*

# Building LR(1) DFA

- *Construct **LR(1) NFA** with all LR(1) items of G as states, connect states by moving the **dot**; then convert the NFA to DFA.*




- *Construct the LR(1) DFA directly (see the Dragon book)*
- *Given a set of LR(1) items* `I`, `CLOSURE(I)` *is now defined as*

```
repeat
  for each item [A -> α.Bβ , a] in I and
      each production B -> γ and each terminal b in FIRST(βa)
                add [B -> .γ, a] to I, if it's not in I
  until I does not change
```

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# Constructing LR(1) Parser

- ***Canonical LR(1) collection** is computed by the following procedure:*

```
I0 = CLOSURE([S' -> .S , $]) and C = {I0}
repeat
  for each I ∈ C and grammar symbol X
    T = GOTO(I,X); if T ≠ ∅ and T ∉ C then C = C ∪ { T };
until C does not change
```

***Resulting** `LR(1) DFA`: `C` **is the set of states;** `GOTO` **is the transition table***

- *From the **LR(1) DFA**, we can construct the parsing table ----- **LR(1) parsing table.** The parser based on LR(1) parsing table is called **LR(1) parser**. The **LR(1) grammars** are those whose LR(1) parsing table does **not** contain any conflicts (no duplicate entries).*
- ***Example:***

```
S' -> S
S -> C C
C -> c C | d
```

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# LALR(1) Parsing

- ***Bad News:** LR(1) parsing tables are too big; for PASCAL, SLR tables has about 100 states, LR table has about 1000 states.*
- ***LALR** (LookAhead-LR) parsing tables have same number of states as SLR, but use lookahead for reductions. The **LALR(1) DFA** can be constructed from the **LR(1) DFA.***
- ***LALR(1) states** can be constructed from LR(1) states by merging states with same **core**, or same LR(0) items, and **union** their lookahead sets.*

```
Merging  I8: C -> cC., c/d          I9: C -> cC., $
into a new state    I89:  C -> cC., c/d/$

Merging  I3: C -> c.C, c/d          I6: C -> c.C, $
             C -> .cC, c/d              C -> .cC, $
             C -> .d, c/d               C -> .d, $
into a new state    I36:  C -> c.C, c/d/$
                          C -> .cC, c/d/$
                          C -> .d, c/d/$I
```

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# LALR(1) Parsing (cont'd)

- *From the **LALR(1) DFA**, we can construct the parsing table ----- **LALR(1) parsing table.** The parser based on LALR(1) parsing table is called **LALR(1) parser**. The **LALR(1) grammars** are those whose LALR(1) parsing table does **not** contain any conflicts (no duplicate entries).*
- *LALR(1) DFA and LALR(1) parsing table can be constructed without creating LR(1) DFA --- see Dragon book for detailed algorithm.*
- ***LALR parser** makes same number of moves as LR parser on **correct input.***
- ***On incorrect input,** LALR parser may make erroneous reductions, but will signal "error" before shifting input, i.e., merging states makes reduce determination "less accurate", but has no effect on shift actions.*

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# Summary: LR Parser

- *Relation of three LR parsers:* ***LR(1) > LALR(1) > SLR(1)***
- *Most programming language constructs are* ***LALR(1)****. The* ***LR(1)*** *is unnecessary in practice, but the* ***SLR(1)*** *is not enough.*
- *YACC is an LALR(1) Parser Generator.*
- *When parsing ambiguious grammars using LR parsers, the parsing table will contain multiple entries. We can specify the* ***precedence*** *and* ***associativity*** *for terminals and productions to resolve the conflicts. YACC uses this trick.*
- ***Other Issues*** *in parser implementation: 1. compact representation of parsing table 2. error recovery and diagnosis.*

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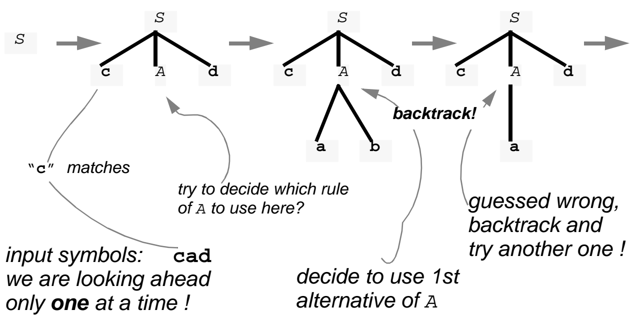
# Top-Down Parsing

- ***Starting from the start symbol*** *and "guessing" which production to use next step. It often uses* ***next input token*** *to guide "guessing".*

example: `S -> c A d`
`A -> ab | a`

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# Top-Down Parsing (cont'd)

- *Typical implementation is to write a* ***recursive procedure*** *for each* ***non-terminal*** *(according to the r.h.s. of each grammar rule)*

***Grammar:***

```
E  -> T E'

E' -> + T E'
    | ε

T  -> F T'

T' -> * F T'
    | ε

F  -> id
    | ( E )
```

`advance` *sets* `c` *to next input token*
`err` *reports error message*

```
fun e() = (t(); eprime())

and eprime() = if (c = "+")
  then (advance(); t(); eprime())

and t() = (f(); tprime())

and tprime() = if (c = "*")
  then (advance(); f(); tprime())

and f() =
  (if (c = id) then advance()
   else if (c = "(") then
     (advance(); e();
      if (c=")") then advance() else err())
     else err()
```

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# Recursive Descent Parsing

- *The previously referred top-down parsing method is often called* ***recursive descent parsing*** *!*
- ***Main challenges:***

1. **back-tracking** is messy, difficult and inefficient
   (solution: use input "lookahead" to help make the right choice)
2. **more alternatives** --- even if we use one lookahead input char, there are still > 1 rules to choose --- `A -> ab | a`
   (**solution**: rewrite the grammar by **left-factoring**)
3. **left-recursion** might cause infinite loop
   what is the procedure for `E -> E + E` ?
   (**solution**: rewrite the grammar by **eliminating left-recursions**)
4. **error handling ---** errors detected "far away" from actual source.

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# Algorithm: Recursive Descent

- ***Parsing Algorithm (using 1-symbol lookahead in the input)***

  1. Given a set of grammar rules for a non-terminal ***A***

     $$A \rightarrow \alpha_1 \mid \alpha_2 \mid \ldots \mid \alpha_n$$

     we choose proper alternative by looking at first symbol it derives ---- the next input symbol decides which $\alpha_i$ we use

  2. for $A \rightarrow \varepsilon$, it is taken when none of the others are selected

- ***Algorithm****: constructing a recursive descent parser for grammar* ***G***

  1. *transform grammer* ***G*** *to* ***G'*** *by removing left-recursions and do the left-factoring.*
  2. *write a (recursive) procedure for each non-terminal in* ***G'***





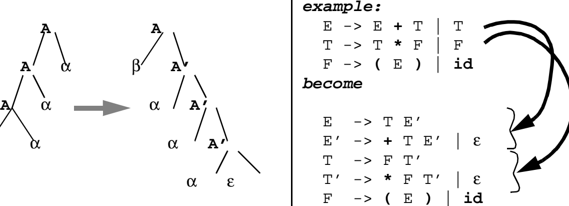
# Left Recursion Elimination

- *Elimination of* ***Left Recursion*** *(useful for top-down parsing only)*

  replace productions of the form $A \rightarrow A\ \alpha \mid \beta$

  with $A \rightarrow \beta A'$
  $A' \rightarrow \alpha\ A' \mid \varepsilon$

  (yields different parse trees but same language)



```
example:
  E -> E + T | T
  T -> T * F | F
  F -> ( E ) | id
become
  E  -> T E'
  E' -> + T E' | ε
  T  -> F T'
  T' -> * F T' | ε
  F  -> ( E ) | id
```

***Important: read Appel pp 51 - 53 for details***





# Left Factoring

- *Some grammars are unsuitable for recursive descent, even if there is no left recursion*

  *"**dangling-else**"*

  ```
  stmt -> if expr then stmt
        | if expr then stmt else stmt
        | ......
  ```

  *input symbol* `if` *does not uniquely determine alternative.*

- ***Left Factoring*** *--- factor out the common prefixes (**see AHU pp 178**)*

  change the production $A \rightarrow x\ y \mid x\ z$

  to $A \rightarrow x\ A'$
  $A' \rightarrow y \mid z$

  ***thus***

  ```
  stmt -> if expr then stmt S'
    S' -> else stmt | ε
  ```





# Predictive Parsing

- ***Predictive parsing*** *is just* ***table-driven*** *recursive descent; it contains:*

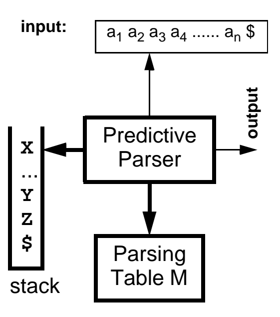
  A **parsing stack** --- contains terminals and non-terminals

  A **parsing table** : a 2-dimensional table **M[X,a]** where **X** is non-terminal, **a** is terminal, and table entries are <u>grammar productions</u> or <u>error indicators</u> .

***<u>algorithm</u>*** *$ **is end-of-file**, S **is start symbol***

```
push($); push(S);

while top <> $ do (
   a <- the input char
   if top is a terminal or $ then
        (if top == a then
              pop(); advance()
         else err())
   else if M[top,a] is X->Y1Y2...Yk then
             (pop();
              push(Yk); ...; push(Y1))
        else err()
)
```



# Constructing Predictive Parser

- *The key is to build the parse table* **M[A,a]**

```
for each production A -> α do
  for each a ∈ FIRST(α) do
        add A -> α to M[A,a]
  if ε ∈ FIRST(α) then
        for each b ∈ FOLLOW(A) do
            add A -> α to M[A,b]

rest of M is error
```

- **FIRST**($\alpha$) *is a set of terminals (plus* $\varepsilon$*) that* ***begin*** *strings derived from* $\alpha$*, where* $\alpha$ *is any string of non-terminals and terminals.*
- **FOLLOW**($A$) *is a set of terminals that can* ***follow*** $A$ *in a sentential form, where* $A$ *is any non-terminal*





# First & Follow

- *To compute* **FIRST(X)** *for any grammar symbol* **X** *:*

**FIRST**(X) = {X}, *if* X *is a terminal;*
**FIRST**(X) = **FIRST**(X) ∪ {a}, *if* X -> aα ;
**FIRST**(X) = **FIRST**(X) ∪ {ε}, *if* X -> ε ; *and*
**FIRST**(X) = **FIRST**(X) ∪ **FIRST**($Y_1Y_2...Y_k$), *if* X -> $Y_1Y_2...Y_k$ .

*repeat until nothing new is added to any* **FIRST**

- **FIRST**($Y_1Y_2...Y_k$) = **FIRST**($Y_1$)-{ε}
∪ **FIRST**($Y_2$)-{ε} *if* ε ∈ **FIRST**($Y_1$)
∪ **FIRST**($Y_3$)-{ε} *if* ε ∈ **FIRST**($Y_1Y_2$)
..................
∪ **FIRST**($Y_k$)-{ε} *if* ε ∈ **FIRST**($Y_1...Y_{k-1}$)
∪ {ε} *if all* **FIRST**($Y_i$)$_{(i=1,...,k)}$ *contain* ε





# First & Follow (cont'd)

- *To compute* **FOLLOW(X)** *for any non-terminal* **X** *:*

**FOLLOW**(S) = **FOLLOW**(S) ∪ {$}, *if* S *is start symbol;*

**FOLLOW**(B) = **FOLLOW**(B) ∪ (**FIRST**(β) - {ε}), *if* A -> α B β *and* β ≠ ε

**FOLLOW**(B) = **FOLLOW**(B) ∪ **FOLLOW**(A) *if* A -> α B *or* A -> α B β *and* ε ∈ **FIRST**(β)

- ***Example:***

| | ***reason*** |
|---|---|
| **FOLLOW**(E) = {**$**}, | E *is start symbol* |
| = {**$ )**}, | F -> **(** E **)** |
| **FOLLOW**(E')= **FOLLOW**(E) | E -> T E' |
| = {**$ )**}, | |

***(Read Appel pp 47 - 53 for detailed examples)***





# Summary: LL(1) Grammars

- *A grammar is* ***LL(1)*** *if parsing table* **M[A,a]** *has no duplicate entries, which is equivalent to specifying that for each production*

**A** -> $\alpha_1$ | $\alpha_2$ | ... | $\alpha_n$

*1. All* **FIRST**($\alpha_i$) *are disjoint.*

*2. At most one* $\alpha_i$ *can derive* ε *; in that case,* **FOLLOW(A)** *must be disjoint from* **FIRST**($\alpha_1$) ∪ **FIRST**($\alpha_2$) ∪ ...... ∪ **FIRST**($\alpha_n$)

- *Left-recursion and ambiguity grammar lead to* ***multiple entries*** *in the parsing table. (try the* ***dangling-else*** *example)*
- *The* ***main difficulty*** *in using (top-down)* ***predicative parsing*** *is in rewriting a grammar into an LL(1) grammar. There is no general rule on how to resolve* ***multiple entries*** *in the parsing table.*