CS 428/528 Lecture 2
Logical Foundations & Coq

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(Slides based on those from the Software Foundations course material developed by Benjamin Pierce at Penn)

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How do we build software?
^ that works
^ (and be convinced that it does)

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Critical Software

Individual programs

- Operating systems
- Network stacks
- Crypto
- Medical devices
- Flight control systems
- Power plants
- Home security
- ...

Programming languages

- Static type systems
- Data abstraction and modularity
- Security controls
- Compiler correctness

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Logic

+ Reasoning about individual programs

+ Reasoning about whole programming languages

SOFTWARE FOUNDATIONS

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LOGICAL FOUNDATIONS

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Q: How do we know something is true?

A: We prove it

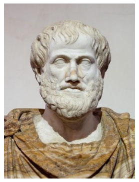
Q: How do we know that we have a *proof*?

A: We need to define what it means for something to be a proof.
A proof is a logical sequence of arguments, starting from some initial assumptions

Q: How do we know that we have a *valid* sequence of arguments? Can any sequence be a proof? E.g.

All humans are mortal
All Greeks are human
Therefore I am a Greek!

A: No, no, no! We need to think harder about valid ways of reasoning…



Aristotle
384 – 322 BC

Euclid
~300 BC

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First we need a language…

- Gottlob Frege: a German mathematician who started in geometry but became interested in logic and foundations of arithmetic.
- 1879 Published "*Begriffsschrift, eine der arithmetischen nachgebildete Formelsprache des reinen Denkens*" (Concept-Script: A Formal Language for Pure Thought Modeled on that of Arithmetic)
  - First rigorous treatment of functions and quantified variables
  - ⊢ A, ¬A, ∀x.F(x)
  - First notation able to express arbitrarily complicated logical statements

Gottlob Frege
1848-1925

BEGRIFFSSCHRIFT,
EINE DER ARITHMETISCHEN NACHGEBILDETE
FORMELSPRACHE
DES REINEN DENKENS.
VON
Dr. GOTTLOB FREGE.
HALLE a/S.
VERLAG VON LOUIS NEBERT.
1879.

Images in this & following slides taken from Wikipedia.

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Formalization of Arithmetic

- 1884: *Die Grundlagen der Arithmetik* (The Foundations of Arithmetic)
- 1893: *Grundgesetze der Arithmetik* (Basic Laws of Arithmetic, Vol. 1)
- 1903: *Grundgesetze der Arithmetik* (Basic Laws of Arithmetic, Vol. 2)
- Frege's goals:
  - isolate logical principles of inference
  - derive laws of arithmetic from first principles
  - set mathematics on a solid foundation of logic

The plot thickens…

Just as Volume 2 was going to print in 1903, Frege received a letter…

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Addendum to Frege's 1903 Book

"Hardly anything more unfortunate can befall a scientific writer than to have one of the foundations of his edifice shaken after the work is finished. This was the position I was placed in by a letter of Mr. Bertrand Russell, just when the printing of this volume was nearing its completion."

– Frege, 1903

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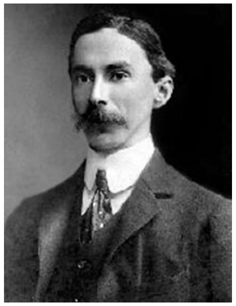
Bertrand Russell

- *Russell's paradox:*

1. Set comprehension notation:
   { x | P(x) } "The set of x such that P(x)"
2. Let X be the set (of sets) { Y | Y ∉ Y }.
3. Ask the logical question:
   Does X ∈ X hold?
4. Paradox! If X ∈ X then X ∉ X.
   If X ∉ X then X ∈ X.



Bertrand Russell
1872 - 1970

- Frege's language could derive Russell's paradox ⇒ it was *inconsistent*.
- Frege's logical system could derive anything. Oops(!!)

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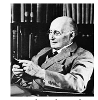
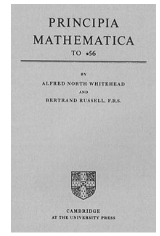
Aftermath of Frege and Russell

- Frege came up with a fix… but it made his logic trivial :-(
- 1908: Russell fixed the inconsistency of Frege's logic by developing a *theory of types*.
- 1910, 1912, 1913, (revised 1927): *Principia Mathematica* (Whitehead & Russell)
  - Goal: axioms and rules from which *all* mathematical truths could be derived.
  - It was a bit unwieldy…

> "From this proposition it will follow, when arithmetical addition has been defined, that 1+1=2."
> —Volume I, 1st edition, *page 379*


Whitehead


Russell



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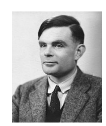
Logic in the 1930s and 1940s

- 1931: Kurt Gödel's first and second incompleteness theorems.
  - Demonstrated that any consistent formal theory capable of expressing arithmetic cannot be complete.
  -
- 1936: Genzen proves consistency of arithmetic.
- 1936: Church introduces the λ-calculus.
- 1936: Turing introduces Turing machines
  - Is there a decision procedure for arithmetic?
  - Answer: no, it's undecidable
  - The famous "halting problem"
    - N.b.: Only in 1938 did Turing get his Ph.D.
- 1940: Church introduces the *simple theory of types*


Kurt Gödel
1906 - 1978


Gerhard Gentzen
1909 - 1945


Alonzo Church
1903 - 1995


Alan Turing
1912 - 1954

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Fast Forward…

- Two logicians in 1958 (Haskell Curry) and 1969 (William Howard) observe a remarkable correspondence:

Haskell Curry
1900 – 1982

William Howard
1926 –

types	~	propositions
programs	~	proofs
computation	~	simplification

N.G. de Bruijn
1918 - 2012

- 1967 – 1980's: N.G. de Bruijn runs Automath project
  - uses the Curry-Howard correspondence for computer-verified mathematics

Basis for modern type systems: OCaml, Haskell, Scala, Java, C#, …

- 1971: Jean-Yves Girard introduces System F
- 1972: Girard introduces Fω
- 1972: Per Martin-Löf introduces intuitionistic type theory
- 1974: John Reynolds independently discovers System F

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… to the Present

- 1984: Coquand and Huet first begin implementing a new theorem prover "Coq"
- 1985: Coquand introduces the calculus of constructions
  - combines features from intuitionistic type theory and Fω
- 1989: Coquand and Paulin extend CoC to the calculus of inductive constructions
  - adds "inductive types" as a primitive
- 1992: Coq ported to Xavier Leroy's OCaml
- 1990's: up to Coq version 6.2
- 2000-2015: up to Coq version 8.4
- 2017: Coq version 8.6

Too many contributors to list here…

- 2013: Coq receives ACM Software System Award

Thiery Coquand
1961 –

Gérard Huet
1947 –

http://coq.inria.fr/refman/Reference-Manual002.html

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So much for foundations… what about the "software" part?

PROGRAMMING (LANGUAGE) FOUNDATIONS

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Building Reliable Software

- Suppose you work at (or run) a software company.
- Suppose, like Frege, you've sunk 30+ person-years into developing the "next big thing":
  - Boeing Dreamliner2 flight controller
  - Autonomous vehicle control software for Nissan
  - Gene therapy DNA tailoring algorithms
  - Super-efficient green-energy power grid controller
- Suppose, like Frege, your company has invested a lot of material resources that are also at stake.
- How do you avoid getting a letter like the one from Russell?

Or, worse yet, *not* getting the letter,
with disastrous consequences down the road?

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Approaches to Software Reliability

- Social
  - Code reviews
  - Extreme/Pair programming
- Methodological
  - Design patterns
  - Test-driven development
  - Version control
  - Bug tracking
- Technological
  - "lint" tools, static analysis
  - Fuzzers, random testing
- Mathematical
  - Sound type systems
  - Formal verification

Less "formal": Lightweight, inexpensive techniques (that may miss problems)

This isn't a tradeoff… all of these methods should be used.

Even the most "formal" argument can still have holes:
- Did you prove the right thing?
- Do your assumptions match reality?

- Knuth: *"Beware of bugs in the above code; I have only proved it correct, not tried it."*

More "formal": eliminate *with certainty* as many problems as possible.

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Can formal methods scale?

Use of formal methods to verify full-scale software systems is a hot research topic!

- CompCert – fully verified C compiler
  Leroy, INRIA
- Vellvm – formalized LLVM IR
  Zdancewic, Penn
- Verified Software Toolchain
  Appel, Princeton
- Bedrock – web programming, packet filters
  Chlipala, MIT
- CertiKOS – certified OS kernel
  Shao, Yale

Vellvm verified LLVM

Verified Software Toolchain

Bedrock

CERTIKOS

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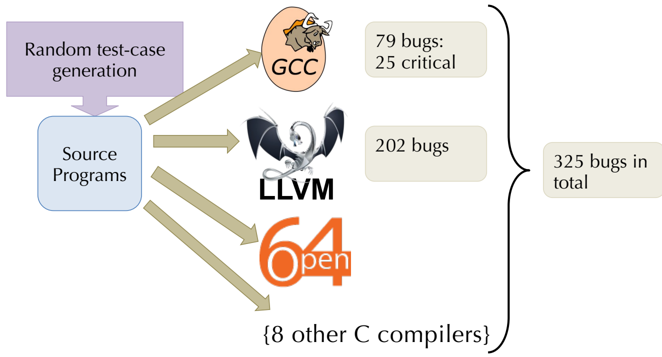
Does it work?

Finding and Understanding Bugs in C Compilers [Yang et al. PLDI 2011]



Verified Compiler: CompCert [Leroy et al.]
<10 bugs found in (at the time *unverified)* front-end component

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Regehr's Group Concludes

The striking thing about our CompCert results is that the *middle-end bugs* we found in all other compilers are *absent*. As of early 2011, the under-development version of *CompCert is the only compiler we have tested for which Csmith cannot find wrong-code errors*. This is not for lack of trying: we have devoted about six CPU-years to the task. *The apparent unbreakability of CompCert supports a strong argument that developing compiler optimizations within a proof framework, where safety checks are explicit and machine-checked, has tangible benefits for compiler users.*

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