Yale

A Compositional Theory of Linearizability

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Yale University, USA

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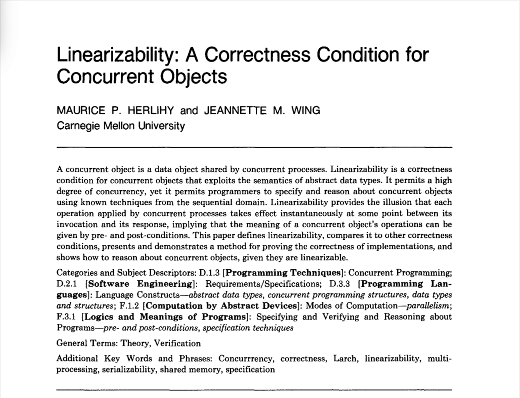
Herlihy-Wing Linearizability



Linearizability: A Correctness Condition for Concurrent Objects

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General Terms: Theory, Verification

Additional Key Words and Phrases: Concurrrency, correctness, Larch, linearizability, multiprocessing, serializability, shared memory, specification

What is a concurrent object?

$$\text{Queue} := \{\text{enq} : \mathbb{N} \to \{\text{ok}\}, \text{deq} : \mathbb{N} + \{\bot\}\}$$

- Sequential:
  deq
- Concurrent:

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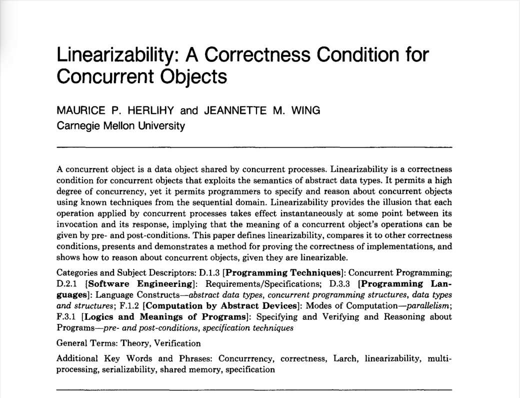

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α_0:deq

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$$\alpha_0\text{:deq} \longrightarrow \alpha_0\text{:}\bot$$

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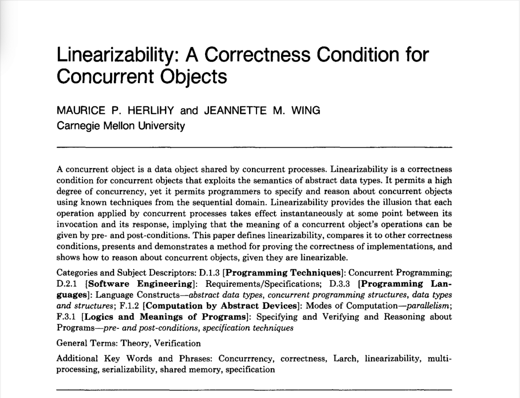

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α_0:deq α_1:enq(1) α_2:enq(2) α_1:ok α_2:ok α_0:$\bot/1/2$

Classical Linearizability: Example

ν'_{squeue} : α_0:deq α_1:enq(1) α_0:1 α_2:enq(2) α_1:ok α_0:deq α_2:ok

α_0:deq → α_0:1 → α_0:deq

α_1:enq(1) → α_1:ok

α_2:enq(2) → α_2:ok

α_0:deq → α_0:1 → α_0:deq

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Classical Linearizability: Example

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---

α_0:deq $\rightarrow$ α_0:1 $\rightarrow$ α_0:deq

α_1:enq(1) $\rightarrow$ α_1:ok

α_2:enq(2) $\rightarrow$ α_2:ok

$\rightsquigarrow$

α_0:deq $\rightarrow$ α_0:1 $\rightarrow$ α_0:deq

α_1:enq(1) $\rightarrow$ α_1:ok

α_2:enq(2) $\rightarrow$ α_2:ok

ν_{squeue} : α_1:enq(1) α_1:ok α_0:deq α_0:1 α_2:enq(2) α_2:ok

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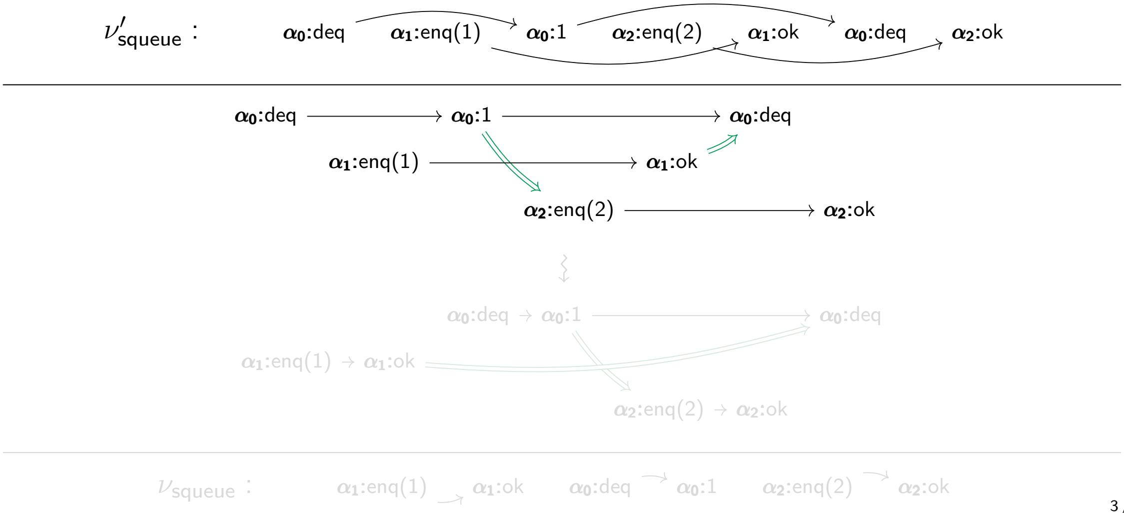

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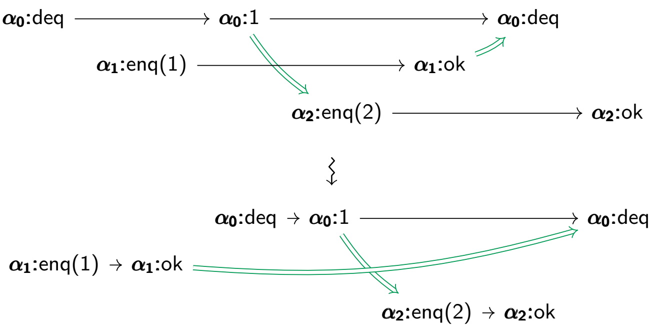

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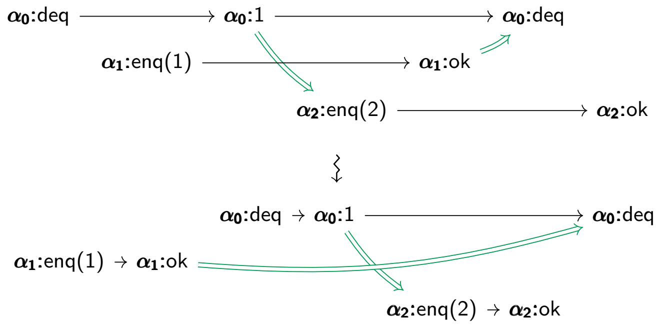
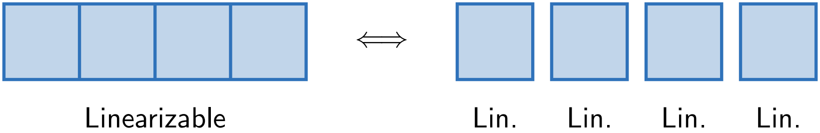


ν_{squeue} : α_1:enq(1) α_1:ok α_0:deq α_0:1 α_2:enq(2) α_2:ok

Locality [Herlihy and Wing, 1990]

PROPOSITION

H is linearizable if and only if, for each object x, $H \mid x$ is linearizable.



Equivalence with Contextual Refinement [Filipović et Al. 2010]

$\mathsf{Obj}_{\mathsf{Conc}}$ observationally refines ($\sqsubseteq$) $\mathsf{Obj}_{\mathsf{Atom}}$ when

$$\forall \text{ programs } P \,.\, \forall \text{ states } s \,.\, [\![P]\!](\mathsf{Obj}_{\mathsf{Conc}})(s) \subseteq [\![P]\!](\mathsf{Obj}_{\mathsf{Atom}})(s)$$

PROPOSITION

$$\mathsf{Obj}_{\mathsf{Conc}} \text{ linearizes to } \mathsf{Obj}_{\mathsf{Atom}} \iff \mathsf{Obj}_{\mathsf{Conc}} \text{ observationally refines } \mathsf{Obj}_{\mathsf{Atom}}$$

Why?

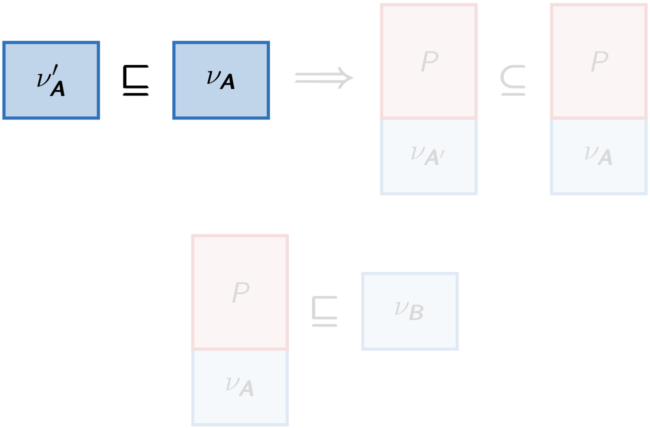
Where does linearizability come from and why does it work?

Key Contributions

- A new generalized definition of linearizability not tied to atomicity.
- The first model of linearizability that supports refinement, horizontal and vertical composition.
- A general (category-theoretic) methodology for deriving linearizability from a model of concurrent computation.
- New simpler proofs of the locality and refinement properties.
- A new program logic that is sound for our formulation of linearizability.
- Applications to compositional verification.

Outline

Typical Approach for Verifying Concurrent Objects



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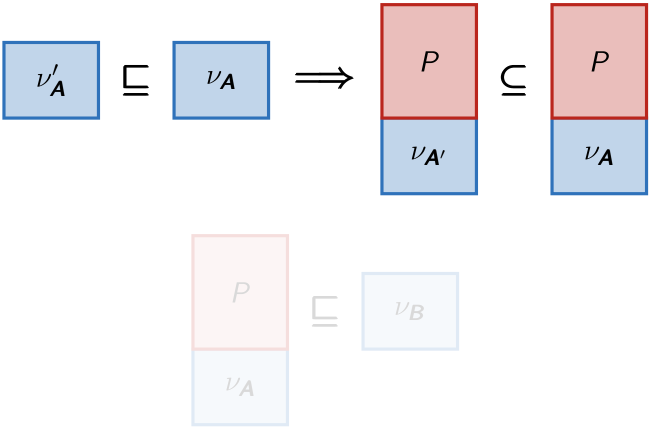

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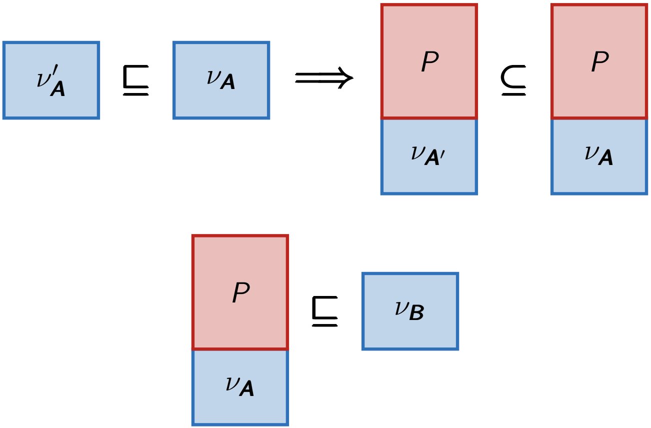
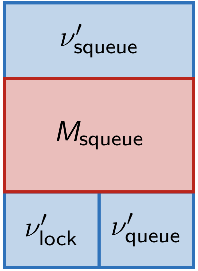


Implementating a Shared Queue

M_{squeue}:

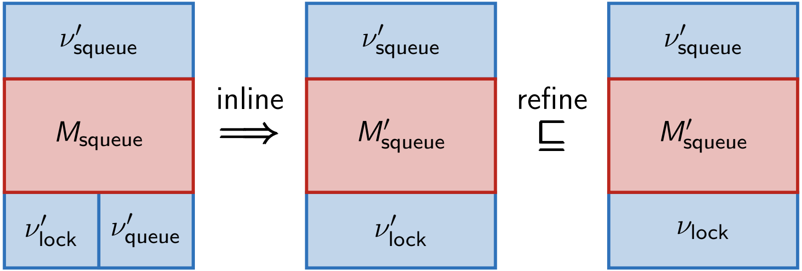
```
Import Q:Queue
Import L:Lock

enq(k) {                deq() {
  L.acq();                    L.acq();
  r <- Q.enq(k);              r <- Q.deq();
  L.rel();                    L.rel();
  ret r                     ret r
}                       }
```

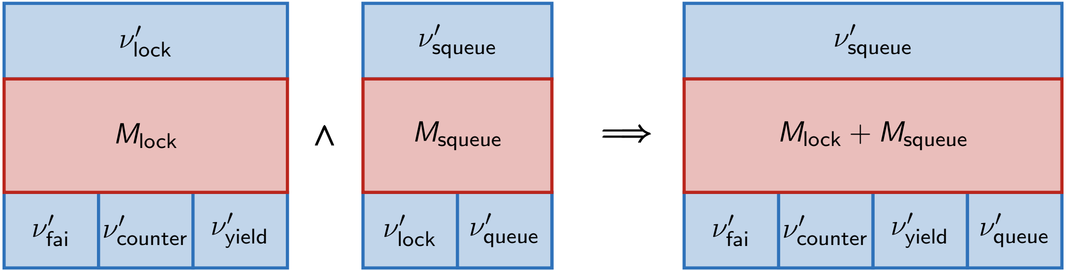


- No account of how locality interacts with refinement.
- Locality doesn't apply! The queue has a race (not linearizable).

Implementing a Shared Queue (Continued)

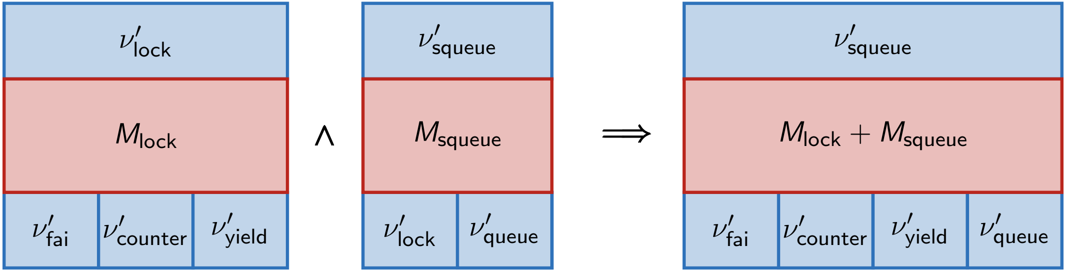


Vertical Composition



Inlining ? Syntactic Linking?

Vertical Composition



Inlining ? Syntactic Linking?

Compositionality

Linearizability	Refinement $- \subseteq -$
Locality $- \otimes -$	(Vertical) Composition $-;-$

Outline

Our Methodology

1. Base Model of Computation
   (A semicategory enriched with a notion of refinement)

2. Choose identity programs
   (Usually obvious)

3. Compute a Compositional Model out of (1) and (2)
   (The Karoubi Envelope)

4. Abstract Linearizability $\iff$ Concrete Linearizability

5. One Extra Axiom $\implies$ Refinement Property

6. Tensor Product + One Extra Axiom $\implies$ Locality

Game Semantics

Types correspond to **Games** A, B, C

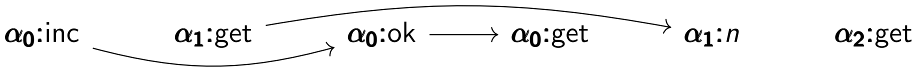
Programs correspond to **strategies** $\sigma : A \multimap B$ of the game $A \multimap B$

Object specifications correspond to strategies $\nu : 1 \multimap A$

Sequentially Consistent Computation

- We start by defining a sequential model of computation.
- A set of agent names $\alpha \in \Upsilon$.
- A concurrent game $\boldsymbol{A}$ is specified by the sequential game A that all agents play.
- A move looks like $\boldsymbol{\alpha}$:m where $\alpha \in \Upsilon$ and m is a move of A.
- The set of plays of $\boldsymbol{A}$ is the set of sequentially consistent interleavings of plays from A.
  Example:

$$\text{Counter} = \{\text{get} : \mathbb{N}, \text{inc} : \text{ok}\}$$



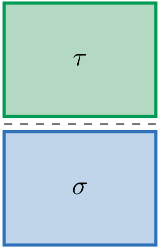
Vertical Composition

There is a composition operation defined per usual by

"Parallel composition + Hiding"

Denoted by

$$\sigma : \boldsymbol{A} \multimap \boldsymbol{B} \qquad \tau : \boldsymbol{B} \multimap \boldsymbol{C} \longmapsto \sigma ; \tau : \boldsymbol{A} \multimap \boldsymbol{C}$$



Which is **associative** ... but there is **no identity element**!

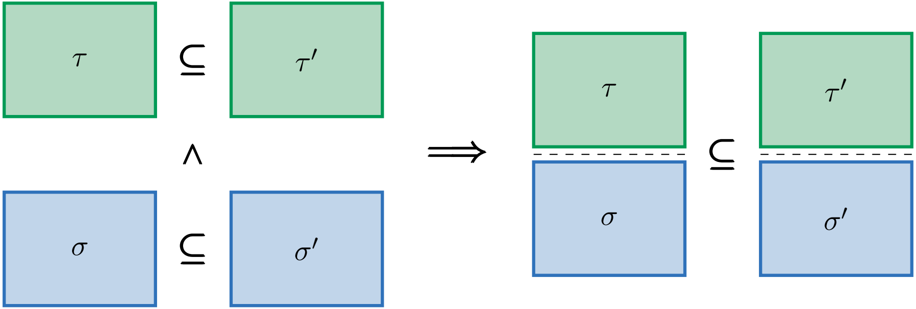
$$\forall \sigma : \boldsymbol{A} \multimap \boldsymbol{B} . \mathbf{id}_{\boldsymbol{A}} ; \sigma ; \mathbf{id}_{\boldsymbol{B}} = \sigma$$

In other words, concurrent games with concurrent strategies assembles into a semicategory

$$\underline{\mathbf{Game}}_{\mathbf{Conc}}$$

Refinement

Our model is enriched over a notion of refinement $\subseteq$ (behavior containment)



Sequential Copycat

```
Import Q:Queue

enq (n : ℕ) {
  r <- Q.enq(n);
  ret r
}

deq () {
  r <- Q.deq();
  ret r
}
```

The copycat strategy $\mathsf{copy}_A : A \multimap A$ behaves as the sequential identity

Concurrent Strategies

$\mathsf{ccopy}_{\boldsymbol{A}} := \|_{\alpha \in \Upsilon} \mathsf{copy}_A$

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‖ ... ‖

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Composition can lead to **emergent behavior**.

$\sigma \subseteq \sigma ; \mathsf{ccopy}_{\boldsymbol{B}}$

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$\|$ $\cdots$ $\|$

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Composition can lead to **emergent behavior**.

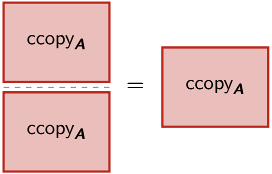
$$\sigma \subseteq \sigma; \text{ccopy}_{\boldsymbol{B}}$$

The Karoubi Envelope

PROPOSITION

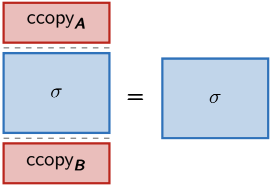
For all concurrent game $\mathbf{A}$ the strategy $\mathsf{ccopy}_{\mathbf{A}} : \mathbf{A} \multimap \mathbf{A}$ is idempotent, i.e.

$$\mathsf{ccopy}_{\mathbf{A}} ; \mathsf{ccopy}_{\mathbf{A}} = \mathsf{ccopy}_{\mathbf{A}}$$



Call a strategy $\sigma : \mathbf{A} \multimap \mathbf{B}$ **saturated** when

$$\mathsf{ccopy}_{\mathbf{A}} ; \sigma ; \mathsf{ccopy}_{\mathbf{B}} = \sigma$$



Composition of saturated strategies is associative and has as identity $\mathsf{ccopy}_{\_}$.
Call the resulting category of concurrent games and saturated strategies

$$\mathbf{Game}_{\mathbf{Conc}}$$

Two Models of Concurrent Computation

$\underline{\mathbf{Game}}_{\mathbf{Conc}}$	$\mathbf{Game}_{\mathbf{Conc}}$
Good for specification	Good for composition

We can convert between models:

$$\sigma : A \multimap B \in \underline{\mathbf{Game}}_{\mathbf{Conc}} \xmapsto{K_{\mathrm{Conc}}-} \mathrm{ccopy}_{A}; \sigma; \mathrm{ccopy}_{B} \in \mathbf{Game}_{\mathbf{Conc}}$$

Outline

Abstract Linearizability

DEFINITION (ABSTRACT LINEARIZABILITY)

We say

$$\nu'_A : A \in \mathbf{Game}_{\mathbf{Conc}}$$

linearizes to

$$\nu_A : A \in \underline{\mathbf{Game}}_{\mathbf{Conc}}$$

when

$$\nu'_A \subseteq K_{\mathsf{Conc}}\ \nu_A$$

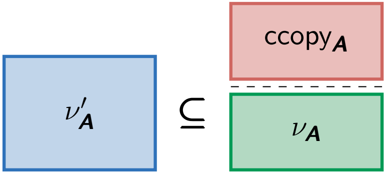
DEFINITION

A linearizable object consists of a pair

$$(\nu'_A : A \in \mathbf{Game}_{\mathbf{Conc}}, \nu_A : A \in \underline{\mathbf{Game}}_{\mathbf{Conc}})$$

such that

$$\nu'_A \subseteq K_{\mathsf{Conc}}\ \nu_A$$



ν'_A is the implementation and ν_A the specification

Rewrites

PROPOSITION (GHICA AND MURAWSKI, 2004)

$\sigma : \boldsymbol{A}$ is saturated if and only if $\forall t \in \sigma. \forall s \in P_{\boldsymbol{A}}. s \rightsquigarrow_{\boldsymbol{A}} t \implies s \in \sigma$

If $t \in \sigma$ and s is "more concurrent" than t then s is also in σ

Linearizability

DEFINITION

$s \in P_{\boldsymbol{A}}$ is linearizable to $t \in P_{\boldsymbol{A}}$ when there exists a sequence s_O of Opponent moves and a sequence s_P of Proponent moves such that

$$s \cdot s_P \rightsquigarrow_{\boldsymbol{A}} t \cdot s_O$$

- t need not be atomic (coincides with Herlihy-Wing when it is);
- s_P = returns;
- s_O = removed pending invocations (not all need be removed);
- $\rightsquigarrow_{\boldsymbol{A}}$ = happens-before order preservation.

Abstract Linearizability

PROPOSITION

Let $\tau : \boldsymbol{A} \in \underline{\textbf{Game}}_{\textbf{Conc}}$ then

$$K_{\mathsf{Conc}}\ \tau = \{s \in P_{\boldsymbol{A}} \mid \exists t \in \tau . s \text{ linearizes to } t\}$$

Abstract Linearizability

PROPOSITION

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COROLLARY

For $\sigma : \boldsymbol{A}$ and $\tau : \boldsymbol{A}$,

$$\sigma \text{ linearizes to } \tau \iff \sigma \subseteq K_{\mathsf{Conc}}\ \tau.$$

Abstract Linearizability

PROPOSITION

Let $\tau : \boldsymbol{A} \in \underline{\textbf{Game}}_{\textbf{Conc}}$ then

$$K_{\mathsf{Conc}}\ \tau = \{s \in P_{\boldsymbol{A}} \mid \exists t \in \tau . s \text{ linearizes to } t\}$$

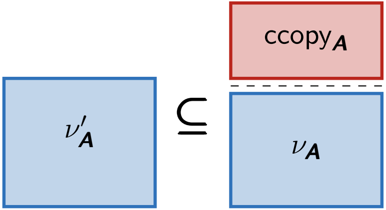
DEFINITION (ABSTRACT LINEARIZABILITY)

We say $\sigma : \boldsymbol{A} \in \textbf{Game}_{\textbf{Conc}}$ linearizes to $\tau : \boldsymbol{A} \in \underline{\textbf{Game}}_{\textbf{Conc}}$ when

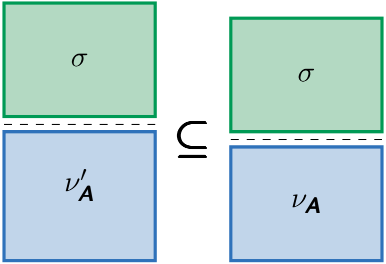
$$\sigma \subseteq K_{\mathsf{Conc}}\ \tau$$

Outline

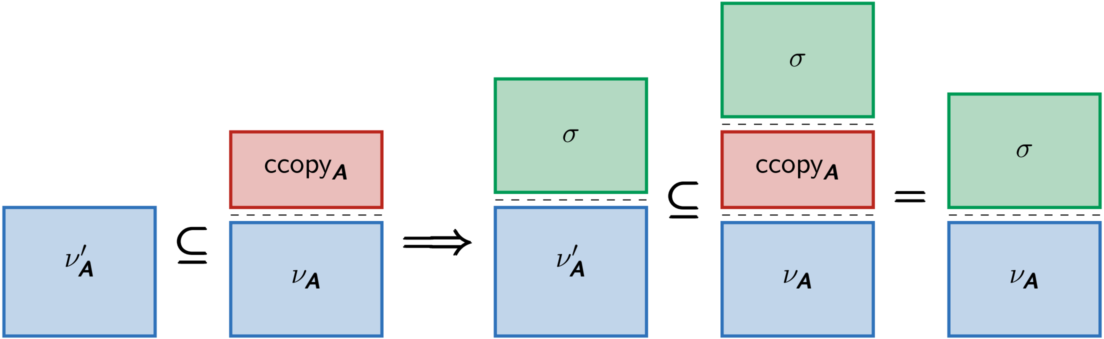
Interaction Refinement



$\iff \forall B. \forall \sigma : A \multimap B.$
$\in \mathbf{Game}_{\mathbf{Conc}}$

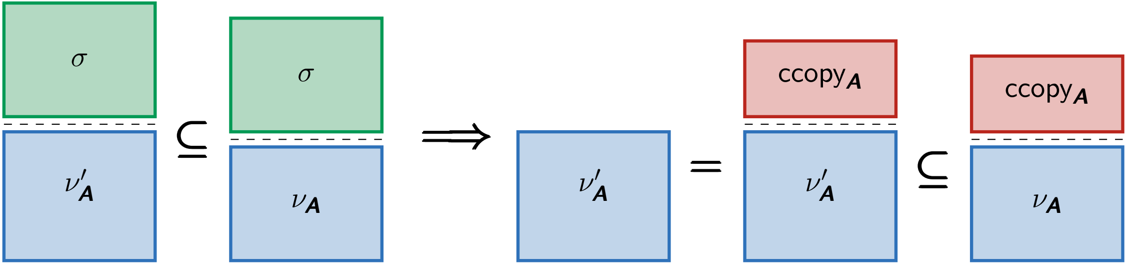


Interaction Refinement: Proof (Forward)



Interaction Refinement: Proof (Backward)

$\forall B. \forall \sigma : A \multimap B.$



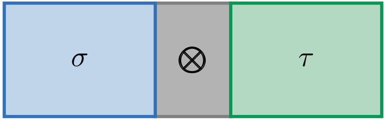
Horizontal Composition

We define a tensor product of strategies:

$$\sigma : \boldsymbol{A} \quad , \quad \tau : \boldsymbol{B} \quad \longmapsto \quad \sigma \otimes \tau : \boldsymbol{A} \otimes \boldsymbol{B}$$

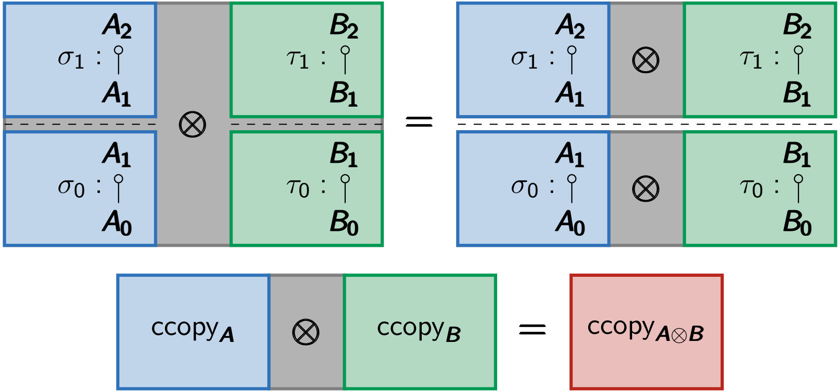
where

$$\sigma \otimes \tau = \text{ all sequentially consistent interleavings of plays of } \sigma \text{ and } \tau$$

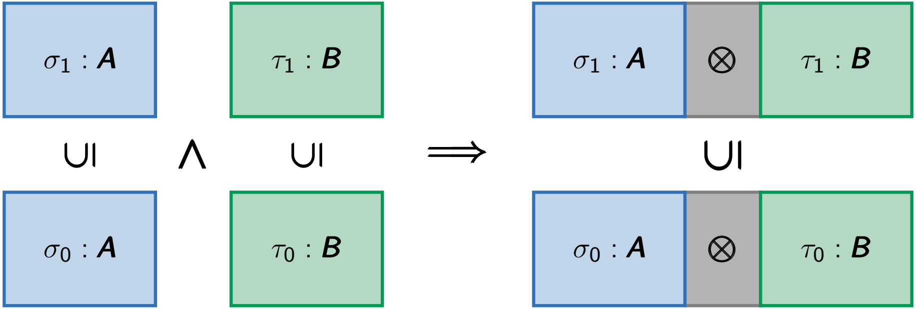


This makes **Game**$_{\mathbf{Conc}}$ into a symmetric monoidal category.
($- \otimes -$ has a unit **1**, is associative and commutative, bifunctor, ...)

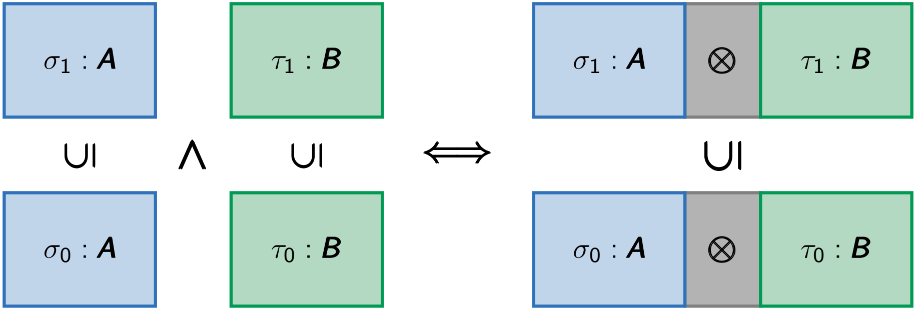
Horizontal Composition: Functorial



Horizontal Composition: Monotonicity

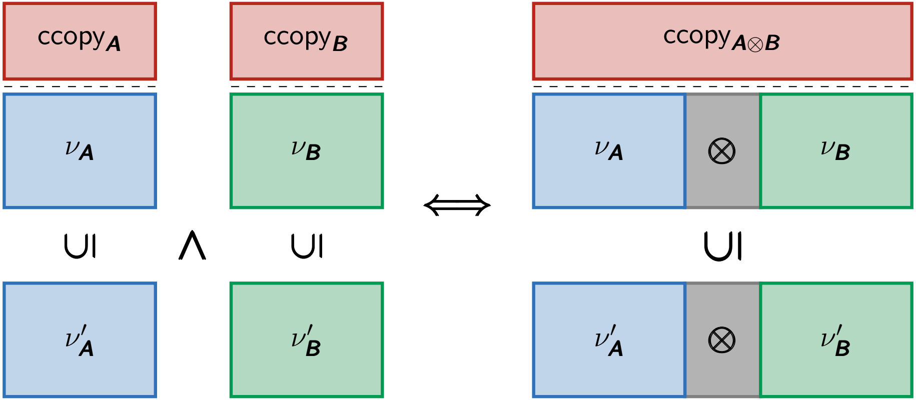


Horizontal Composition: Order-Isomorphism

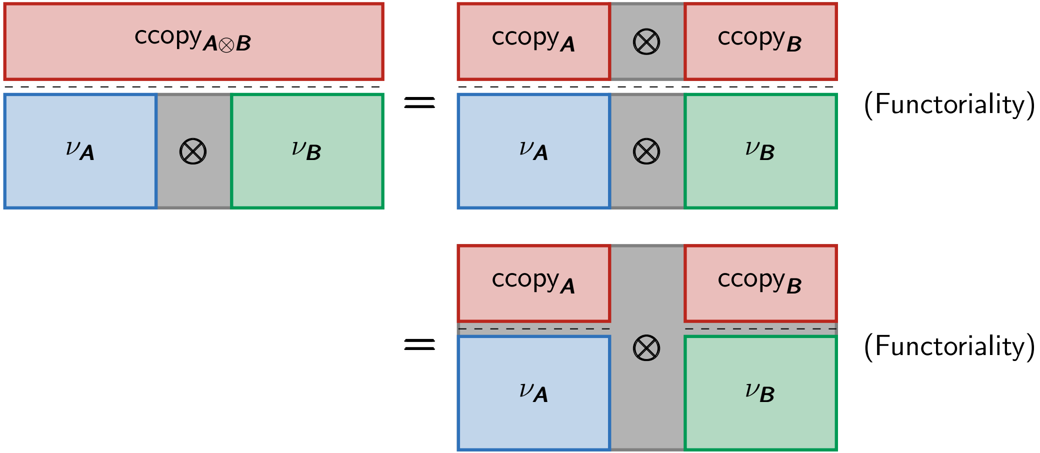


Locality

THEOREM

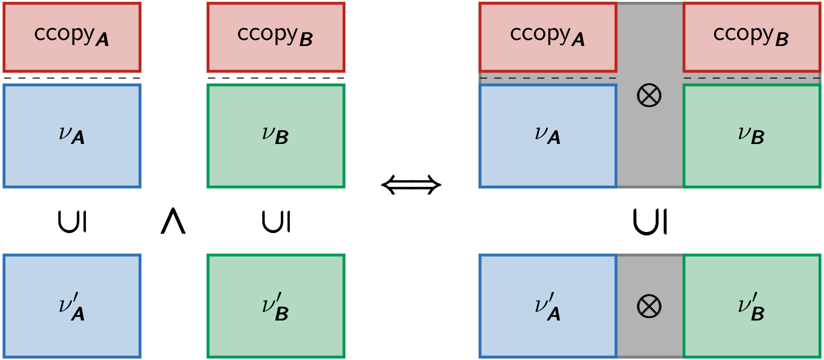


Locality: Proof



Locality: Proof

THEOREM



Holds by the order-isomorphism

Locality Proof [Herlihy and Wing, 1990]

Let $<$ be the transitive closure of the union of all $<_x$ with $<_H$. It is immediate from the construction that $<$ satisfies Conditions (1) and (2), but it remains to be shown that $<$ is a partial order. We argue by contradiction. If not, then there exists a set of operations $e_1, \ldots, e_n$, such that $e_1 < e_2 < \cdots < e_n$, $e_n < e_1$, and each pair is directly related by some $<_x$ or by $<_H$. Choose a cycle whose length is minimal.

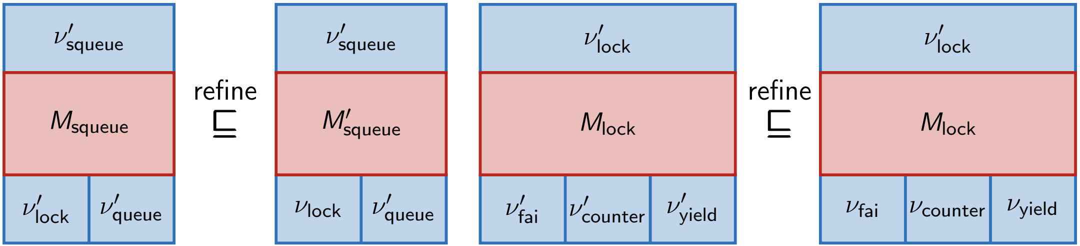
Suppose all operations are associated with the same object x. Since $<_x$ is a total order, there must exist two operations e_{i-1} and e_i such that $e_{i-1} <_H e_i$ and $e_i <_x e_{i-1}$, contradicting the linearizability of x.

The cycle must therefore include operations of at least two objects. By reindexing if necessary, let e_1 and e_2 be operations of distinct objects. Let x be the object associated with e_1. We claim that none of $e_2, \ldots, e_n$ can be an operation of x. The claim holds for e_2 by construction. Let e_i be the first operation in $e_3, \ldots, e_n$ associated with x. Since e_{i-1} and e_i are unrelated by $<_x$, they must be related by $<_H$; hence the response of e_{i-1} precedes the invocation of e_i. The invocation of e_2 precedes the response of e_{i-1}, since otherwise $e_{i-1} <_H e_2$, yielding the shorter cycle $e_2, \ldots, e_{i-1}$. Finally, the response of e_1 precedes the invocation of e_2, since $e_1 <_H e_2$ by construction. It follows that the response to e_1 precedes the invocation of e_i, hence $e_1 <_H e_i$, yielding the shorter cycle $e_1, e_i, \ldots, e_n$.

Since e_n is not an operation of x, but $e_n < e_1$, it follows that $e_n <_H e_1$. But $e_1 <_H e_2$ by construction, and because $<_H$ is transitive, $e_n <_H e_2$, yielding the shorter cycle $e_2, \ldots, e_n$, the final contradiction. □

Outline

Implementing a Shared Queue



Program Logic

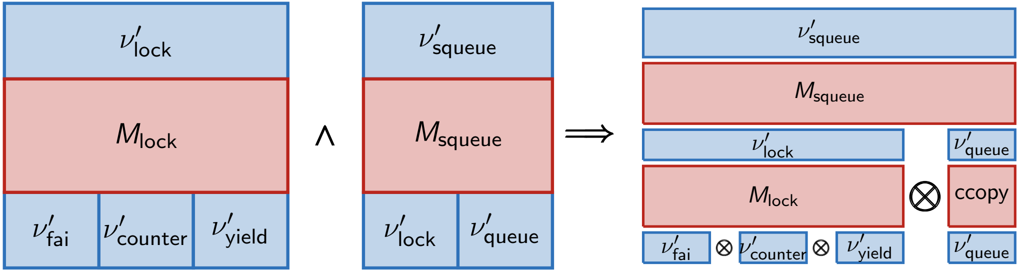
- We define a program logic for showing individual programs implement linearizable objects.
- Sound for our notion of linearizability (and in particular for, interval-linearizability).
- Directly connects with our compositional theory.

PROPOSITION (SOUNDNESS)

If $\mathcal{R}[A], \mathcal{G}[A] \models_A \{P[A]\}\ M[A]\ \{Q[A]\}$ and $(\nu'_E : \dagger\boldsymbol{E}, \nu_E : \dagger\boldsymbol{E})$ is a linearizable concurrent object then

$$\nu'_E ; [\![M[A]]\!] \cap \nu'_F \subseteq K_{\mathsf{Conc}}\ \nu_F$$

Composing Verified Components



Conclusion

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- New foundations for linearizability and its properties.
- A compositional theory for linearizability.
- Promising applications for compositional verification.

Check our paper and TR for more:

- The concurrent game semantics model
- The category-theoretic axiomatization
- Thorough comparison with previous work
- The example we described in this talk
- Full program logic description
- More...

Thank you!

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